Abstract

We present in this article a condensed review of the rigorous and often forgotten proof of the inconsistency of Black Hole Thermodynamics and the Holographic Principle in Quantum Gravity, by following very closely the author’s self-published critical book [1] on quantum black holes. Most of the important results discussed here follow directly from the classic articles by Harms and Leblanc [2, 3, 4, 5, 6].
1 Introduction

A *classical* black hole is a very high density topological object warping spacetime so dramatically that all matter and radiation crossing its horizon become trapped forever. Because light itself cannot escape from this horizon, the hole is black for external observers. But what laws of physics rule when the black hole horizon becomes so small that its size approaches the scale of the Compton wavelength is the fundamental question at the core of Quantum Gravity (QG) research today.

In the seventies, Bekenstein [7, 8, 9] approached the problem from the viewpoint of information theory and drew analogies between the behavior of a black hole horizon area with that of entropy in Thermodynamics.

On the other hand, Stephen Hawking [10, 11, 12] and others [13, 14, 15, 16], worked on the black hole problem making use of semiclassical approaches. In particular, Hartle and Hawking [13] and Gibbons and Hawking [14] quantized General Relativity (GR) with a black hole metric by making use of the Euclidean (imaginary time) path integral formalism in the semiclassical WKB approximation. Although a consistent quantum theory of gravity remained elusive at that time, especially due to problems of non renormalizability of quantum General Relativity, they went ahead anyway with their semiclassical calculations, assuming somehow that they were valid.

The WKB approximation of the Euclidean path integral of Quantum Gravity in the black hole instanton sector was interpreted by Hartle and Hawking [13] and also Gibbons and Hawking [14, 15] as describing the thermal (canonical) partition function of a gas of quantum particles in the background of a black hole playing the role of a thermal reservoir at the Bekenstein-Hawking temperature, a temperature related to the black hole mass. In this way Bekenstein’s concept of black hole entropy was effectively linked with semiclassical theory. In natural units ($\hbar = c = G = k_B = 1$), Bekenstein-Hawking entropy was found to be one quarter of the black hole horizon surface, a result known as the black hole *area law*. This WKB theory, now known as Black Hole Thermodynamics (BHTD), therefore described a single quantum black hole as a heat reservoir in thermal equilibrium with its radiation.

Hawking apparently confirmed this viewpoint by making use of a mean-field theory in which he quantized particles around a classical black hole background [10, 11, 12]. He found that particles radiated (tunnelled) out of the black hole at exactly the Bekenstein-Hawking temperature. Therefore, contrary to classical black holes, quantum black holes of the Bekenstein-Hawking type allowed particles to escape across the classical horizon with a thermal signature. The thermal radiation from the black hole was called the *Hawking black hole evaporation effect*, and was quite similar to the so-called *Unruh effect* [17, 18] describing thermal particle emission at the Davies-Unruh temperature from a flat spacetime vacuum, as seen by constantly accelerated observers. The analogy between the Unruh and Hawking effects was viewed as a “quantized” form of the Equivalence Principle [19, 20, 21, 22].

Quantum black holes being thermal objects in Hawking’s theory, their equilibrium state is then characterized by the Bekenstein-Hawking canonical entropy. Now this entropy (the area law) constitutes the fundamental basis for the so-called *Holographic Principle* [23] constraining the amount of information (in bits per square meter) contained in a theory of Quantum Gravity. This
principle claims to provide a universal link between geometry (horizon) and information (entropy).

What was found then appeared quite revolutionary at the time. By trying to include quantum effects into black hole spacetimes making use of semiclassical methods, the above authors astonishingly ended up with a theory where pure states in fact could evolve into mixed thermal states characterized by the Bekenstein-Hawking (Davies-Unruh) canonical temperature, thereby violating the laws of Quantum Theory. This unorthodox situation was called the black hole Information Loss Paradox, given the link between entropy and information, or lack thereof.

Let us now briefly review the semiclassical basis of Black Hole Thermoodynamics. In Quantum Statistical Mechanics, the grand canonical partition function $\Xi(\beta, \mu, V)$ of a system in thermal equilibrium (assuming its existence) is given by the well known trace formula,

$$
\Xi(\beta, \mu, V) = \text{Tr}[\mathcal{P} e^{-\beta(\mathcal{H} - \mu \mathcal{N})}],
$$

(1.1)

where $\mathcal{H}$ and $\mathcal{N}$ are the Hamiltonian and number operators respectively, with the trace constrained to be carried out over the physical states of the system, which is sometimes implemented with the help of a suitable projection operator $\mathcal{P}$. Here $\beta$ represents the inverse temperature and $\mu$ the chemical potential of the system.

In Quantum Field Theory, since the works of Matsubara and others in the nineteen fifties [24, 25, 26], it has been customary to explicitly calculate the partition function of various physical systems by a technique called the Matsubara imaginary time method, which consists in replacing real time by imaginary time in the expression of field operators, their Green functions [27, 28] as well as the integration variables of the total action. In the path integral formalism [29, 30, 31], the trace formula Eq. (1.1) is evaluated as the following Feynman-Kac formula for a collection of fields $\{\phi_i\}$,

$$
\Xi(\beta, \mu, V) = \int \prod_i [d\phi_i] [\mathcal{P}_i] e^{-S_E[\phi_i]/\hbar},
$$

(1.2)

where $S_E[\phi_i]$ is the Euclidean action evaluated over a finite interval $[0, \beta h]$ of Euclidean time $\tau$,

$$
S_E[\phi_i] = \int_0^{\beta h} d\tau L_E[\phi_i(\tau)],
$$

(1.3)

and where $\beta$ is interpreted as the inverse canonical temperature of the system.

In the case of gravity, the path integral is carried over the Euclidean space-time metric tensor field and further divided into the topologically distinct sectors determined from the classical part of the spacetime geometry.

Evaluation of the path integral (1.2) can be carried out explicitly in the semiclassical WKB approximation where the dominant (saddle point) field configurations $\phi_c^i$ coincide with the minimum of the classical Euclidean action. These are instanton field configurations which are solutions of the classical Euclidean equations of motion. As a result, the path integral (1.2) can also be interpreted as the decay probability of the false metastable instanton vacuum [32, 33, 34]. We therefore have,

$$
\Xi(\beta, \mu, V) \simeq e^{-S_E[\phi_c^i]/\hbar}[1 + O(h)] \simeq P[\phi_c^i],
$$

(1.4)
where the on-shell Euclidean action term $S_E[\phi_c]/\hbar$ plays the role of a “thermo-
dynamic potential” in the thermal interpretation and $P[\phi_c]$ represents the decay
probability of the false vacuum in the alternative purely quantum interpretation.
The terms represented by $O(\hbar)$ originate formally from field theoretical loop cor-
rections which, following widespread standard considerations, are assumed to
be well-defined. We thus have two possible physical interpretations for the same
WKB expression.

In the context of General Relativity with black holes or black objects, the
interpretation of the path integral as a canonical partition function constitutes
the fundamental assumption of the Bekenstein-Hawking Black Hole Thermo-
dynamics (BHTD) formalism, while the competing interpretation as the decay
probability of the false black hole vacuum constitutes the fundamental assump-
tion of the pure state WKB Leblanc-Harms theory (LHT) [1, 2, 3, 4, 5, 6]. Let
us now take a closer look at Black Hole Thermodynamics.

2 Black Hole Thermodynamics

In $D$ dimensions, following the considerations of Gibbons and Hawking [14],
it is therefore assumed that the grand canonical partition function of a gas of
gravitons and matter fields in a stationary black hole background in the WKB
approximation is given by Eq. (1.4) with $\phi_c$ describing a stationary Euclidean
and asymptotically flat black hole geometry, periodic in Euclidean time with
instanton period $\beta\hbar$. Since the partition function is the generating function
of all thermodynamical quantities, the whole of Black Hole Thermodynamics
should be readily derived from it.

Problems arise immediately however with these calculations since local theo-
ries of quantum gravity are known not to be renormalizable. Therefore Eq. (1.4)
is in fact meaningless in its original context, unless it originates from some closed
string or $p$-brane theories, which are non-local and ultraviolet finite quantum
theories of gravity. We are assuming of course that they form fully consistent
theories themselves, which seems to require the presence of supersymmetry.

Notwithstanding these remarks, riding along the popularly accepted proce-
dure, we shall go ahead with the standard interpretation of Eq. (1.4) as the
grand canonical partition function of a gas of gravitons in a one black hole-
instanton background playing the role of a heat bath. Unfortunately, as will be
seen later on, this will lead us to fatal inconsistencies.

We nevertheless proceed by considering the general case of a $D$-dimensional
stationary and spherically symmetric geometry. Its real-time line element can
be written as follows (we set $c = 1$ from now on),

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2d\Omega_{D-2}^2,$$

where $\Phi(r)$ and $\Lambda(r)$ are functions of the radial coordinate only and $d\Omega_{D-2}$ is
the line element of the unit $(D-2)$-sphere,

$$d\Omega_{D-2}^2 = \sum_{n=1}^{D-2} \prod_{k=1}^{n-1} \sin^2 \theta_k d\theta_n^2.$$

The analytic continuation of the above line element to imaginary time $i\tau$ is
given by,

$$ds^2 = e^{2\Phi(r)}d\tau^2 + e^{2\Lambda(r)}dr^2 + r^2d\Omega_{D-2}^2.$$
To connect with stationary black hole geometry, we further assume that the functions $e^{\Phi(r)}$ and $e^{\Lambda(r)}$ are positive definite for $r > r_+$, where $r_+$ is the location of an outer event horizon for which the Euclidean time-time matrix element of the metric tensor satisfies the following relation,

$$e^{2\Phi(r_+)} = 0.$$  \hfill (2.4)

Expanding the Euclidean geometry near the event horizon $r = r_+$, Eq. (2.3) can be re-written approximately as follows,

$$ds^2 \simeq \left[ (e^{-\Lambda(r)} \partial_r e^{\Phi(r)}) \right]_{r=r_+}^2 R^2 d\tau^2 + dR^2 + r_+^2 d\Omega_{D-2}^2.$$  \hfill (2.5)

The Euclidean time $\tau$ is readily seen to play the role of an angular variable in the $(R, \tau)$-plane and the requirement of the vanishing of the conical singularity at $R(r_+) = 0$ further constrains it to be periodic with a period $\beta \bar{h}$ given by the following formula,

$$\kappa \equiv [e^{-\Lambda(r)} \partial_r e^{\Phi(r)}]_{r=r_+} = \frac{2\pi}{\beta \bar{h}},$$ \hfill (2.6)

where $\kappa$ is the so-called surface gravity of the black hole. Equation (2.6) constitutes the celebrated result found by Hawking relating the black hole surface gravity to the Bekenstein-Hawking canonical temperature $\beta^{-1}$. When the metric is specified in terms of the black hole mass and its various conserved charges (quantum hair), such an equation further provides a relationship between these physical parameters and the black hole canonical temperature.

Let us now turn to the explicit calculation of the Euclidean action $S_E$ in order to find the grand canonical partition function (1.4) for black holes. This calculation is non-trivial as it involves, in addition to the Einstein-Hilbert and matter actions, surface terms originating from the extrinsic curvature of the boundary of spacetime.

The $D$-dimensional bare Euclidean action for a metric $g$ and matter fields $\phi$ is then given as follows,

$$S^{(\text{bare})}_E[g, \phi] = -\int_{M_D} \left[ \frac{R[g]}{16\pi G_D} + L_{\text{matter}}[g, \phi] \right] - \oint_{\partial M(\infty)} \left[ \frac{K}{8\pi G_D} \right],$$ \hfill (2.7)

where $R$ is the scalar curvature, $L_{\text{matter}}$ is the matter Lagrangian, $K$ is the trace of the extrinsic curvature of the manifold’s boundary $\partial M(\infty)$ at spatial infinity and $G_D$ is Newton’s constant in $D$ dimensions. The (Gibbons-Hawking) surface term proportional to the extrinsic curvature is required in order for the variational principle to be properly defined. The field equations are shown to be satisfied under the condition that the induced surface metric and fields are held fixed on the boundary.

For spatially compact geometries, the Euclidean action (2.7) is finite, but diverges for the non-compact cases. According to Hawking and Horowitz [16], analogously to defining the zero-point of the energy, the physical action must be defined relative to a reference background geometry. Such a “renormalized” action is therefore defined as follows,

$$S_E[g, \phi] \equiv S^{(\text{bare})}_E[g, \phi] - S^{(\text{bare})}_E[g_0, \phi_0],$$ \hfill (2.8)

in which the fields $(g, \phi)$ approach the reference background $(g_0, \phi_0)$ as the surface boundary recedes to spatial infinity ($r \to \infty$).
For asymptotically flat geometries, the reference background is chosen to be flat spacetime with no matter fields. Consequently we get,

\[ S_E[g, \varphi] = - \int_M \left[ \frac{R[g]}{16\pi G_D} + L_{\text{matter}}[g, \varphi] \right] - \oint_{\partial M(\infty)} \left[ K - K_0 \right] \frac{8\pi G_D}{8}. \quad (2.9) \]

Equation (2.9) is the standard expression traditionally used to evaluate the Euclidean action of the various black hole geometries. This equation reproduces the earlier prescription by Gibbons and Hawking [14], which subtracts the action of a flat spacetime with same induced geometry at the boundary.

Let us therefore evaluate explicitly such a reputed “physical” boundary action for the general \( D \)-dimensional stationary black hole metric given by Eq. (2.3). To this end, we first evaluate the contribution from the boundary at finite radius \( r \). We will then let the boundary recede to infinity at the end of the calculation. There are two equivalent approaches: The Coleman-Preskill-Wilzcek method \[35, 36\] and the Kallosh-Ortin-Peet method \[37\].

Following the work of Kallosh, Ortin and Peet \[37\], itself based on considerations by Gibbons and Hawking \[14\], the boundary action in Eq. (2.9) at finite \( r \) can be derived as well by explicitly calculating the trace of the extrinsic curvature for the Euclidean metric (2.3). One finds,

\[ K(r) = \frac{r^{2-D}}{\sqrt{g_{\tau\tau} g_{rr}}} \partial_r [\sqrt{g_{\tau\tau}} r^{D-2}], \quad (2.10) \]

for the curved spacetime metric contribution and,

\[ K_0(r) = r^{2-D} \partial_r [r^{D-2}], \quad (2.11) \]

for the flat space reference background contribution, where,

\[ g_{\tau\tau} = e^{2\Phi(r)}; \quad g_{rr} = e^{2\Lambda(r)}. \quad (2.12) \]

The boundary action in Eq. (2.9) at finite \( r \) is now suggestively written as follows,

\[ S_{bd}^{(r)} = - \frac{1}{\kappa} \frac{A_D(r)}{4G_D} \left[ \kappa(r) + \frac{(D - 2)\sqrt{g_{\tau\tau}}}{r} \left( \frac{1}{\sqrt{g_{rr}}} - 1 \right) \right], \quad (2.13) \]

where \( A_D(r) \) is the area of a \((D - 2)\)-dimensional spherical boundary surface at radius \( r \),

\[ A_D(r) \equiv V_{D-2} r^{D-2}; \quad V_{D-2} \equiv \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})}, \quad (2.14) \]

with \( V_{D-2} \) the volume of the unit \((D - 2)\)-sphere, and \( \kappa(r) \) is the surface gravity,

\[ \kappa(r) \equiv \frac{1}{2\sqrt{g_{\tau\tau} g_{rr}}} \partial_r g_{\tau\tau}. \quad (2.15) \]

We remark that,

\[ \kappa(r_+) = \kappa, \quad (2.16) \]

which is the surface gravity at the horizon, related to the black hole canonical temperature through formula (2.6).
Recalling Eqs. (1.4) and (2.9), we therefore write the following expression for the grand canonical partition function of the black hole system,

\[-\ln \Xi(\beta, \mu, V) = \left[ S_{\text{Einstein-Hilbert}} + S_{\text{matter}} + S_{\text{bd}}^{(\infty)} \right]/\hbar = S_E/\hbar. \tag{2.17}\]

Again following the work of Kallosh, Ortin and Peet [37], it is found that the mean value of the energy can be calculated within the path integral formalism in terms of the Euclidean action evaluated this time by taking into account the fact that constant Euclidean time surfaces have two boundaries, one at \( r \to \infty \) and another one at the black hole event horizon \( r = r_+ \). For a propagation during the time interval \( \tau_2 - \tau_1 = \beta \), the mean energy is expressed as follows,

\[ \beta(\bar{E} - \mu \bar{N}) = \left[ S_{\text{Einstein-Hilbert}} + S_{\text{matter}} + S_{\text{bd}}^{(\infty)} - S_{\text{bd}}^{(r_+)} \right]/\hbar = \left[ S_E - S_{\text{bd}}^{(r_+)} \right]/\hbar. \tag{2.18}\]

Combining Eqs. (2.17) and (2.18), it is now straightforward to calculate the grand canonical entropy \( S(\beta, \mu, V) \) of the black hole system,

\[ \frac{1}{k_B} S(\beta, \mu, V) = \ln \Xi(\beta, \mu, V) + \beta(\bar{E} - \mu \bar{N}), \tag{2.19}\]

one finds immediately,

\[ \frac{1}{k_B} S(\beta, \mu, V) = -\frac{1}{\hbar} S_{\text{bd}}^{(r_+)}. \tag{2.20}\]

Therefore the entropy is solely determined from the boundary term at the black hole event horizon.

Making use of equations (2.12), (2.13), (2.16) as well as (2.4), we arrive at the well known formula,

\[ \frac{1}{k_B} S(\beta, \mu, V) = \frac{A_D(r_+)}{4G_D}\hbar , \tag{2.21}\]

which is Hawking’s celebrated area law for the entropy.

Let us now take a closer look at the specific and simple example of the \( D \)-dimensional non-rotating neutral Schwarzschild black hole.

The \( D \)-dimensional Schwarzschild black hole is the spherically symmetric non-rotating solution of Einstein’s equations in the vacuum. \( L_{\text{matter}} \) vanishes in Eq. (2.9) and so does the chemical potential term. Consequently, the Schwarzschild case is described in BHTD as a simple canonical ensemble. Einstein’s equations are then written as follows,

\[ R_{\mu\nu} = 0. \tag{2.22}\]

In terms of the functions parametrizing the general spherically symmetric metric (2.3), the solution is given explicitly as follows,

\[ e^{2\Phi(r)} = e^{-2\Lambda(r)} = 1 - \left( \frac{r_+}{r} \right)^{D-3}, \tag{2.23}\]

where \( r_+ \) is the black hole event horizon.
At such a minimum of the action, the Einstein-Hilbert term in expression (2.9) vanishes. Therefore the Euclidean periodic Schwarzschild instanton contribution to the action originates solely from the boundary term and we arrive at the following expression for the Euclidean action of the $D$-dimensional Schwarzschild black hole,

$$
S_E(r_+) = \frac{2\pi^{\frac{D-1}{2}}}{16\pi G_D \Gamma\left(\frac{D-1}{2}\right)} \beta h r_+^{D-3}, \quad (2.24)
$$

where, recalling the formula (2.6) for the instanton period, $\beta h$ is explicitly evaluated as follows,

$$
\beta h = \frac{4\pi r_+}{(D-3)}, \quad (2.25)
$$

leading to the following expression for the black hole Euclidean action,

$$
S_E(r_+) = \frac{2\pi^{\frac{D-1}{2}}}{4G_D \Gamma\left(\frac{D-1}{2}\right)} \frac{r_+^{D-2}}{(D-3)} = \frac{1}{(D-3)} \frac{A_D(r_+)}{4G_D}. \quad (2.26)
$$

Having calculated the Euclidean action in terms of the horizon radius, and so as a function of the Bekenstein-Hawking temperature $\beta^{-1}$, we are now ready to derive the relationships between the horizon radius $r_+$ and the black hole mass $M$. To that effect, let us recall the expression for the canonical mean (internal) energy $\bar{E}(\beta)$,

$$
\bar{E}(\beta) \equiv \frac{\partial}{\partial \beta} [\beta F(\beta)], \quad (2.27)
$$

where $\beta F(\beta)$ is the Helmholtz free energy given by,

$$
\beta F(\beta) = \frac{S_E(r_+(\beta))}{h}. \quad (2.28)
$$

This relation comes from the fact that the thermodynamic potential of Eq. (2.17) for the grand canonical ensemble corresponds to the canonical partition function in the limit of vanishing chemical potential. Now the mean energy of the system should be identified with the black hole mass $M$. Recalling Eq. (2.25) for the temperature, the defining equation for the mean energy can be re-written as follows,

$$
M = \frac{(D - 3)}{4\pi} \frac{\partial [S_E(r_+)]}{\partial r_+}. \quad (2.29)
$$

Making use of Eq. (2.26) for the Euclidean action, we get,

$$
M = \frac{(D - 2)V_{D-2}}{16\pi G_D} r_+^{D-3}, \quad (2.30)
$$

from which one verifies that Newton’s gravitational law is recovered in the large $r$ limit of the time-time matrix element of the metric tensor. We then have,

$$
\beta M = \frac{(D - 2)A_D(r_+)}{(D - 3) 4G_D \bar{h}}. \quad (2.31)
$$

We are now in a position to calculate the canonical entropy of the system. Recalling the basic formula for the entropy,

$$
\frac{1}{k_B} S(\beta) = \beta M - \beta F(\beta), \quad (2.32)
$$
and making use of equations (2.26), (2.28) and (2.31), we find as expected the well known result,

\[ S(\beta) = \frac{k_B A_D(r_+)}{4G_D \hbar} \tag{2.33} \]

which is Hawking’s area law of Eq. (2.21). Note that the entropy (2.33) and the Euclidean action (2.26) coincide only in 4 dimensions. In terms of the black hole mass, the entropy is expressed as follows,

\[ S(M) = k_B \sigma(D) M^{\frac{D-2}{D-3}} \tag{2.34} \]

while the expression (2.25) for the temperature is re-written as,

\[ \beta(M) = \frac{(D-2)}{(D-3)} \sigma(D) M^{\frac{1}{D-3}} \tag{2.35} \]

where,

\[ \sigma(D) \equiv \frac{4\pi \bar{h}}{h(D-3)} \left[ \frac{16\pi G_D}{V_{D-2}(D-2)^{\frac{D-2}{D-3}}} \right]^{\frac{D-3}{D}} \tag{2.36} \]

If we now assume the equivalence of the canonical and microcanonical ensembles, a natural requirement if thermal equilibrium is truly realized in this system, one would be led to the following exponentially rising form for the black hole statistical mechanical density of states \( \Omega(M) \) for mass levels between \( M \) and \( M + \delta M \),

\[ \Omega(M) \delta M = e^{S(M)/k_B} \approx e^{\sigma(D) M^{\frac{D-2}{D-3}}} \tag{2.37} \]

In 4 dimensions, equation (2.37) yields the fastest growing density of states known to physics. It is given explicitly as follows,

\[ \Omega(M) \delta M \approx e^{4\pi G M^2/\hbar} \quad (D = 4) \tag{2.38} \]

Finally, making use of the expressions (2.27)-(2.28) for the mean energy \( \bar{E}(\beta) \) of the system, the canonical heat capacity is readily calculated as follows,

\[ C_V(\beta) \equiv \frac{\partial \bar{E}}{\partial T} = -k_B \beta^2 \frac{\partial \bar{E}}{\partial \beta} = -(D-2)S(\beta) \tag{2.39} \]

which is negative. In 4 dimensions we recover the standard result,

\[ C_V(\beta) = -\hbar k_B \frac{\beta^2}{8\pi G} \quad (D = 4) \tag{2.40} \]

### 3 Inconsistency of Black Hole Thermodynamics

The canonical partition function is the generating function of the canonical ensemble. Once such a function is known for a given system, all of its thermodynamical attributes can readily be calculated. Expressed as an energy integral, the partition function corresponds to the Laplace transform of the statistical mechanical (microcanonical) density of states \[38, 39\],

\[ Z(\beta, N, V) = \int_0^\infty dE e^{-\beta E} \Omega(E, N, V) \]

\[ = \int_0^\infty dE/\delta E \, e^{-[\beta E - S(E, N, V)/k_B]} \tag{3.1} \]
Now the integrand in Eq. (3.1) has a sharp peak at a value $E^*$ which represents the most probable value of the energy at equilibrium. It is situated at the minimum of the argument of the exponential and so we can expand the integrand in this neighborhood,

$$Z(\beta,N,V) \sim \int_{0}^{\infty} \frac{dE}{\delta E} e^{-\frac{\beta E^* - S(E^*,N,V)}{k_B} - \frac{k_B \beta^2}{2C_V} (E - E^*)^2 + \ldots} ,$$

(3.2)
in which we made use of the usual relationships between the heat capacity and the fluctuation term,

$$C_V \equiv \left( \frac{\partial E}{\partial T} \right)_{N,V} ,$$

(3.3) and,

$$\frac{1}{k_B} \left( \frac{\partial^2 S}{\partial E^2} \right)_N = -\frac{k_B \beta^2}{C_V} .$$

(3.4)

The most probable value for the energy is now evaluated from the following condition,

$$\beta E^* - S(E^*,N,V)/k_B = \min \iff \beta = \frac{1}{k_B} \left( \frac{\partial S}{\partial E} \right)_{E=E^*} ,$$

(3.5)
together with the crucial stability condition,

$$\left( \frac{\partial^2 S}{\partial E^2} \right)_{E=E^*} < 0 \iff C_V > 0 .$$

(3.6)

As we can see, the positivity of the heat capacity $C_V$ is a necessary condition for the integral representations (3.1)-(3.2) to exist. Neglecting the higher order fluctuation terms, we can evaluate the Gaussian integral and we arrive at the following expression relating the canonical partition function to the entropy at the most probable value $E^*$,

$$Z(\beta,N,V) \sim \sqrt{\frac{2\pi C_V/k_B}{\beta \delta E}} e^{-\frac{\beta E^* - S(E^*,N,V)/k_B}{k_B}} ,$$

(3.7)

(C_V > 0).

Again the requirement of positivity for the heat capacity is explicit in the above equation. A negative heat capacity would imply the non-existence of the canonical ensemble description, i.e. thermal equilibrium could not be achieved physically.

A positive heat capacity means that, if one injects energy into a system, then its temperature will increase accordingly. For systems in thermal equilibrium, heat flows from hot to cold, so that the injection of heat energy will warm up the system.

Thermal equilibrium systems belong to the class of so-called normal systems. Equilibrium is achieved usually because the interaction among the constituents is short-range and falls off sufficiently rapidly outside the volume containing the system. These systems are commonly found in condensed matter physics and chemistry, as well as in our everyday lives. For them, the thermodynamic limit does exist and the probability distribution satisfies the equipartition theorem.

Besides the class of normal systems, there is yet another class not so well known and called small systems. Examples of small systems are found in nuclear multifragmentation [40, 41] as well as first order phase transitions with
interphase surface boundaries. For such systems, the thermodynamic limit does not exist and the equilibrium is usually far from thermal equilibrium. They can be studied only within the fundamental *microcanonical ensemble* approach which parametrizes the system in terms of the physically conserved quantities such as energy, particle number, angular momentum, charge, etc..

Other examples of small systems can be found in cases with long range interactions such as the gravitational and Coulomb interactions. Such interactions do not fall off sufficiently rapidly at spatial infinity and are still sizable outside the system boundaries. All gravitational systems, including black holes, fall into this category.

Let us now prove that for Black Hole Thermodynamics, the canonical and microcanonical ensembles *are not equivalent* and so the theory cannot describe systems classified as *normal* (thermal), for which the thermodynamical limit does exist. It is in fact a completely inconsistent theory. We shall concentrate on the 4-dimensional Schwarzschild black hole case with mass $M$, but the general conclusion remains exactly the same for all cases of black holes and black branes in higher dimensions.

In BHTD, as seen in the Introduction, one studies imaginary (Euclidean) time black hole solutions to Einstein equations (gravitational instantons). Imaginary time is interpreted as the inverse canonical temperature, following the well known Matsubara imaginary time formalism (trick) of Quantum Field Theory at finite temperature [24, 25, 26]. The corresponding path integral in the semiclassical (WKB) approximation yields the canonical partition function for the black hole. Recalling Eq. (2.28), we find a finite value for the Hawking canonical partition function,

$$ Z_{\text{Hawking}} = e^{-\beta F(\beta)} \sim e^{-S_E/\hbar} = \text{finite}, $$

where $F(\beta)$ is the Helmholtz free energy and $S_E$ is the 4-dimensional black hole (instanton) Euclidean action evaluated in section 2 as $(D = 4)$,

$$ S_E/\hbar = 4\pi GM^2/\hbar. $$

The last expression on the right-hand-side (rhs) of Equation (3.8) is obtained from the semiclassical WKB approximation of the gravitational field integral of Eq. (1.4) evaluated in the (here Schwarzschild) instanton sector with zero chemical potential. The instanton period (inverse temperature) is related to the surface gravity $\kappa$ at the horizon, and is found to be,

$$ h\beta = 2\pi/\kappa = 8\pi GM. $$

The canonical entropy has also been found in section 2 $(D = 4)$,

$$ S(\beta)/k_B = \beta \bar{E} - \beta F(\beta) = S_E/\hbar = \frac{A_4}{4G\hbar}, $$

where the mean energy $\bar{E}$ is identified with the black hole mass $M$ and the last equality is the 4-dimensional area law. Note that the equality between the black hole entropy and the Euclidean action remains valid only in 4 dimensions. At higher dimensions, discrepancies start to occur as is readily seen by comparing Eqs. (2.26) and (2.33), although the entropy always satisfies the area law.
Assuming now the self-consistency of the theory, we should have equality of the canonical and microcanonical entropy since both ensembles are supposed to be equivalent for thermal equilibrium. We are then led to the following exponentially rising form for the black hole statistical mechanical density of states at mass level $M$,

$$\Omega(M)\delta M = e^{S(M)/k_B} \approx e^{4\pi GM^2/\hbar}.$$  \hspace{1cm} (3.12)

Unfortunately, such a rapidly growing density of states is profoundly at odds with the fundamentals of the Quantum Statistical Mechanics of normal systems. Most important of all, we know from Eqs. (2.39)-(2.40) that the black hole canonical heat capacity is negative,

$$C_V(\beta) \equiv \partial \bar{E}/\partial T = -k_B\beta^2 \partial E/\partial \beta = -\hbar k_B\beta^2/8\pi G,$$  \hspace{1cm} (3.13)

where again the mean energy $\bar{E}$ is the black hole mass $M$.

As discussed earlier, the problem of course is that there is no such a thing as a negative canonical heat capacity. Thermal equilibrium can solely exist in systems for which the heat capacity is positive and consequently the finite result (3.8) for the Bekenstein-Hawking canonical partition function must be wrong. It is a trivial mathematical exercise to prove it. The canonical partition function is the Laplace transform of the density of state $\Omega(M)$. Making use of the above explicit expression (3.12) for the black hole density of states, we get immediately,

$$Z(\beta) = \int_0^\infty dE/\delta E \exp[-\beta E + 4\pi G E^2/\hbar] \to \infty, \ \ (\forall \beta),$$  \hspace{1cm} (3.14)

which is a devastating result for Bekenstein-Hawking’s Black Hole Thermodynamics because the black hole canonical partition function is infinite for all temperatures. One easily shows that the divergence comes from the negative heat capacity. Let us expand Eq. (3.14) about the would-be saddle point of Eq. (3.10) as follows,

$$\ln Z(\beta) = \ln Z_{\text{Hawking}}(\beta) + \ln \int_0^\infty dE/\delta E \exp[-(k_B\beta^2/2C_V)(E-M)^2 + \ldots] \to \infty, \ \ (C_V < 0).$$  \hspace{1cm} (3.15)

Obviously the Gaussian integral (the fluctuation part) is divergent for negative heat capacity. Consequently the above integral does not have a saddle point at Eq. (3.10) and the canonical and microcanonical ensembles are not equivalent. Thermal equilibrium does not exist at any temperature in this system. Equations (3.8)-(3.13) are consequently meaningless and we are compelled to abandon completely the interpretation of the imaginary time variable in the line element of Eqs. (2.4) and (2.5) as being related to the canonical temperature of the black hole system.

The semiclassical WKB approximation of the path integral of Eq. (1.4) in the gravitational instanton sector does not describe the thermodynamics of any physical system.

We shall see in the next section how the physical meaning of this WKB calculation can be rescued by properly interpreting the meaning of imaginary time for what it really is, i.e. the gravitational instanton period.
Note that the above conclusions also hold true for black holes or black branes in $D$ dimensions \[1, 2, 3, 4, 5, 6\], where the leading order argument of the exponential behaves as $E^{(D-2)/(D-3)}$ instead of $E^2$. The existence of a positive heat capacity domain in the extremality region of some of these theories (e.g. the dilaton and Reissner-Nordström black holes) does not invalidate these conclusions as it is the high energy behavior that is responsible for the infinities encountered. Here again the partition functions diverge for all temperatures and all black hole or black brane theories do not make any sense as finite temperature theories \[1, 2, 3, 4, 5, 6\]. This completes the proof of the total inconsistency of Bekenstein-Hawking Black Hole Thermodynamics.

Now since Hawking’s entropy (the area law) constitutes the fundamental basis for the so-called Holographic Principle [23] constraining the amount of information (in bits per square meter) contained in a theory of Quantum Gravity, we immediately conclude such a principle to be invalid. The fall of Black Hole Thermodynamics therefore implies that there is no universal link between geometry (horizon) and information (entropy) and that the Holographic Principle cannot and does not constitute a proper conceptual foundation for a consistent theory of Quantum Gravity.

The above considerations also apply to all gravitational systems! Thermal equilibrium can never be achieved for systems with long range forces, i.e. forces with a range greater than the size of the system. For such physical systems, the thermodynamical limit simply does not exist. Black holes therefore are not normal thermodynamical systems. A gas of black holes falls into the category of so-called small systems, which can only be studied within the context of the fundamental microcanonical ensemble \[1, 2, 3, 4, 5, 6\], along, for instance, with nuclear fragmentation systems as well as systems undergoing first order phase transition with interphase surface tension. For such cases, entropy is a generally non extensive global quantity given by Boltzmann’s fundamental formula (the “Einstein Principle”),

$$S(E, \delta E) = k_B \ln[\Omega(E)\delta E].$$

(3.16)

But to study such a gas of black holes, one must first find out what kind of objects quantum black holes really are. This is what we address in the following section.

4 The Leblanc-Harms WKB Theory

Recalling Eq. (1.4), let us go back to the double interpretation of the semiclassical WKB approximation of the gravitational Euclidean path integral in the black hole instanton sector.

The traditional and popular interpretation in terms of the thermodynamical partition function in the grand canonical ensemble at the Bekenstein-Hawking temperature leads directly to Black Hole Thermodynamics. However, as we saw in the previous section, such an interpretation leads to fatal inconsistencies and consequently, the instanton period $\beta\hbar$ cannot represent an inverse canonical temperature.

The only alternative available to us, if any meaning at all must be given to the WKB formula (1.4), is therefore the one commonly used in field theory, particle physics and string theory, namely the probability of decay of the false
metastable vacuum state of the system to a global stable vacuum. Therefore the Euclidean action in the instanton sector contributes an imaginary part (effective potential) of the full quantum theory.

While the corresponding imaginary parts of the free energy or thermodynamic potential are interpreted at finite temperature as yielding half the decay rate of the metastable phase [34, 42, 43, 44, 45, 46], they are interpreted at zero temperature as giving half the decay rate per unit volume of the false vacuum [32, 33, 47, 48, 49].

In our case, this interpretation of course is purely quantum mechanical in nature and does not involve any statistical mechanical concepts such as entropy. The false black hole semiclassical vacuum state being a highly degenerate metastable ground state, it should decay into the less degenerate and more stable no-black hole vacuum state ($M = 0$). By the Principle of Equal Weights (PEW), which applies equally well to both Statistical and Quantum Mechanics, the number of open decay channels at mass level $M$ should be inversely proportional to the semiclassical decay probability $\rho(M)$. Now since the number of open channels grows in parallel with the black hole quantum degeneracy of states (level density) $\rho(M)$ as energy is increased [2], we therefore arrive at the following fundamental expression for the quantum degeneracy of states of semiclassical black holes in the WKB approximation [1, 2, 3, 4, 5, 6],

\[ \rho(M) \propto e^{S_E(M)/\hbar} \left[ 1 + O(\hbar) \right] \approx P^{-1}(M), \tag{4.1} \]

where $P(M)$ represents the decay probability of the false vacuum and terms of the order of $O(\hbar)$ represent field theoretical loop corrections. Eq. (4.1) constitutes the fundamental postulate of the Leblanc-Harms pure state semiclassical WKB theory (LHT) of quantum black holes [1, 2, 3, 4, 5, 6].

The above equation is strikingly similar to the usual expression for the microcanonical density of states at energy level $M$,

\[ \Omega(M)\delta M = e^{S(M,\delta M)/kB} = W(M,\delta M), \tag{4.2} \]

where $S(M,\delta M)$ and $W(M,\delta M)$ are the microcanonical entropy and statistical weight of the system respectively. There are major differences however. First of all, for arbitrary dimensions $D$, the arguments of the exponential do not match in both Eqs. (4.1) and (4.2). One is expressed in terms of the Euclidean action $S_E(M)$ while the other is given by the entropy $S(M,\delta M)$. These quantities are not generally the same. As we saw in section [2] going back to the BHTD theory, they would only coincide for the $D = 4$ case. Secondly they do not represent the same physical quantities. The quantum degeneracy of states represents the exact number of fundamental quantum states at a given energy level, while the statistical weight represents the number of states in the mesoscopic or macroscopic observational window given by the interval $[M, M + \delta M]$. No entropy concept whatsoever enters in the Leblanc-Harms theory.

We now proceed to identify quantum black holes from their degeneracy of states. Let us re-write the fundamental postulate of LHT expressed by Eq. (4.1) as follows,

\[ \rho_{BH}(M;D) \approx c(M;D)e^{S_E(M,D)/\hbar}, \tag{4.3} \]

where the prefactor $c(M;D)$ represents loop corrections to the dominant semiclassical exponential term originating from the instanton action.
Recalling Eq. (3.9) for the 4-dimensional Schwarzschild black hole Euclidean action, we find the following explicit expression for the degeneracy of states,

$$\rho_{BH4}(M) \simeq c(M)e^{4\pi GM^2/\hbar},$$

(4.4)

which should be compared with the softer behavior of the degeneracy of states of quantum string excitations at high mass level $M$ [50],

$$\rho_{\text{string}}(M) \simeq c(M_0/M)^a e^{bM},$$

(4.5)

in which the constants $c$, $a$ and $b$ are model-dependent and where $b^{-1}$ is identified with the so-called Hagedorn temperature of the string system. As for $M_0$, it represents an infrared mass level threshold below which the string spectrum is viewed as discrete.

Generalizing the discussion to the $D$-dimensional cases by recalling Eqs. (2.26) and (2.33)-(2.36), and neglecting the effects of quantum hair [1, 2, 3, 4, 5, 6], we arrive at the following leading behavior for the degeneracy of states of general semiclassical black holes as a function of the mass $M$,

$$\rho_{BH}(M; D) \simeq c(M; D) \exp\left\{\bar{\sigma}(D) M^{\frac{D-2}{2D-3}}\right\},$$

(4.6)

where we defined,

$$\bar{\sigma}(D) \equiv \frac{\sigma(D)}{D-3},$$

(4.7)

and $\sigma(D)$ has been defined in Eq. (2.36).

The above degeneracy of states applies equally well to black $(10-D)$-branes in 10 dimensions. Note that in this case $M$ represents the mass density per unit $(10-D)$-volume, with $D$ constrained as $4 \leq D \leq 10$ [51, 52].

Now calculations of the degeneracy of states of higher dimensional structures such as quantum $p$-branes have been presented during the nineteen seventies by a number of authors [53, 54, 55] and more recently by Alvarez and Ortin [56]. Further considerations by Bytsenko and collaborators [57] have sought to determine more precisely the shape of these degeneracies by including calculations of the prefactors from loop effects.

So the asymptotic behavior of the degeneracy of states of quantum $p$-branes at large mass $M$ is generically given as follows,

$$\rho_{\text{brane}}(M; p) \simeq d(M; p) \exp\left\{\gamma(p) M^{\frac{2p}{D+1}}\right\},$$

(4.8)

where $p$ is the dimensionality of the extended objects. Such an asymptotic behavior holds in arbitrary spacetime dimensions.

One is then very much tempted to compare both the degeneracy of states (4.6) for black holes or black objects, and the above Eq. (4.8) for $p$-branes. Equating the leading behavior in the mass parameter $M$, one then arrives at the following identification of quantum black holes in $D$ dimensions and black $(10-D)$-branes in 10 dimensions,

$$p = \frac{D - 2}{D - 4}.$$  

(4.9)

For black $(10-D)$-branes, only three solutions exist for integer $p$ in the allowed range $4 \leq D \leq 10$. They are $p = 2$ ($D = 6$), $p = 3$ ($D = 5$) and $p = \infty$. 

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We then conclude that the above black 4-branes in 10 dimensions are 2-branes, the black 5-branes in 10 dimensions are 3-branes and finally the black 6-branes in 10 dimensions are ∞-branes.

Analogously, we find that 6-dimensional neutral black holes are 2-branes, 5-dimensional neutral black holes are 3-branes and 4-dimensional neutral black holes are ∞-branes. On the other hand ∞-dimensional neutral black holes are strings (1-branes).

Overall, what these identifications fundamentally show is that, very basically, quantum black holes or black objects are Planck scale resonances \[1, 58, 59\], because they can be identified with p-branes massive excitations.

Note that if we take the view of quantum strings or p-branes as being consistent quantum gravity theories, we come to realize that general relativistic black holes or black objects can become doubly modified, once through higher derivative (non-local) classical contributions arising fundamentally from the intrinsic non local nature of strings and p-branes, and once from purely quantum theoretic (loop) corrections, the so-called back-reaction effects \[60\].

Both the non local and loop effects modify the Euclidean action, and consequently the exact shape of the degeneracy of states as a function of the black hole mass undergoes some modifications. In particular, it may very well be sensible to expect that quantum gravity effects will induce an effective cosmological constant in the original metric as loop corrections induce a non-zero vacuum energy. Consequently, it may very well be that there is no such a thing as a quantum Schwarzschild black hole. The metric instead would be of the Schwarzschild-de Sitter type, which contains an event as well as a larger cosmological horizon. This quantum-induced change of the topology of the black hole spacetime may have further effects on the exact shape of the black hole degeneracy of states. However, although the identifications given by Eq. (4.9) may not be accurate in a final analysis, quantum black holes are still clearly described as resonant excitations of highly non local quantum objects, although their identity may be more difficult to determine accurately.

Finally, let us show that the Leblanc-Harms theory presented here is in conceptual agreement with standard discussions on Planck scale black holes \[1\].

As classical solutions to Einstein’s field equations, black holes present us with a special challenge when confronted with Quantum Theory. Black holes are topological objects endowed with an event horizon, a causal surface trapping all forms of matter and radiation which traverse it. Beyond that point, there is classically no way to escape the fall toward the central singularity. The question arises as to what happens when the black hole horizon radius approaches its Compton wavelength, while its mass increases simultaneously toward the Planck mass scale. Let us define the Schwarzschild and “effective” Compton radii of a static neutral Schwarzschild black hole of mass \(M\) as follows,

\[
R_G \equiv \frac{2GM}{c^2}, \quad (4.10)
\]

\[
R_C \equiv 2\lambda_C = \frac{2\hbar}{Mc}, \quad (4.11)
\]

where \(\lambda_C\) is the corresponding Compton wavelength of the black hole, \(G\) is Newton’s gravitational constant in four spacetime dimensions and \(c\) represents as usual the speed of light in the vacuum. Please note that the effective Compton radius is defined above in such a way that it is equal to the Schwarzschild radius
at the Planck scale. This accounts for the factor of two in its definition. Taking the ratio, we get,

\[ \frac{R_G}{R_C} = \frac{M^2}{M_P^2} \rightarrow 1, \quad (M \rightarrow M_P), \tag{4.12} \]

where \(M_P\) is the Planck mass,

\[ M_P \equiv \sqrt{\frac{\hbar c}{G}}. \tag{4.13} \]

The corresponding Planck length \(l_P\) is nothing but the Compton wavelength of a Planck mass particle,

\[ l_P = \frac{\hbar}{M_P c} = \sqrt{\frac{G\hbar}{c^3}}. \tag{4.14} \]

At such a microscopic scale, quantum fluctuations represented by the Compton wavelength are very important and of the same order of magnitude as the classical horizon radius. Therefore, the very concept of a classical horizon, and thus the classical concept of a black hole itself, becomes problematic. At that scale, General Relativity ceases to be valid and some kind of Quantum Field Theory (\(M\)-Theory?) should take over.

It is argued in fact that all particle excitations with a mass greater than the Planck mass should be viewed as quantum black holes, because their effective Compton radius lies inside their Schwarzschild radius. Taking this property as a precise definition of microscopic black holes and because it is the case for string and \(p\)-brane massive resonant excitations, we arrive directly at the fundamental result of the Leblanc-Harms WKB theory (LHT) \[1, 2, 3, 4, 5, 6\].

Therefore true quantum black holes or black objects correspond indeed to Planck scale quantum excitations similar to nuclear resonances, with no classical horizon. The only difference lies in the mass scale. Nuclear resonances are not quantum black holes because the hadron mass scale is much lower than the Planck mass scale. Their effective Compton radii are consequently much larger than their gravitational Schwarzschild radii, unlike string and \(p\)-brane resonances.

Now because of their finite size, quantum resonances (whether nuclear or string based) are unstable particles and so should not appear as asymptotic states in traditional Hilbert space scattering theory, if one is to insist on the unitarity of the \(S\)-matrix.

The proper field theoretical formulation of a quantum theory of unstable particles remains an outstanding problem in theoretical physics and seems to awaken deep and difficult issues such as fundamental irreversibility at the quantum level. In string and \(p\)-brane theories, the problem fundamentally shows up as a tachyon in the bosonic sector. Supersymmetry (SUSY) is just hiding it under the carpet, but the problem still persists and shows up again as soon as supersymmetry is broken. Interesting avenues have been proposed however, among which quantization in rigged Hilbert space (RHS) \[61, 62\] seems the most promising.

One additional question which arises now is the following: How can one properly recover macroscopic black holes with an horizon structure in the classical limit of string or \(p\)-brane theories. This is a subject of research involving the so-called trans-Planckian (also called ultra-Planckian) theories which try
to understand horizon or closed trapped surface formation from string theory scattering processes in the eikonal approximation. The Italian works of Amati, Ciafaloni and Veneziano \cite{63, 64, 65, 66, 67, 68}, Ciafaloni and Colferai \cite{69}, Veneziano and Wosiek \cite{70, 71}, and Veneziano \cite{72}, as well as the American contributions from Giddings \cite{73, 74} and also Giddings, Gross and Maharana \cite{75} are among the most cited references on this subject.

In this context, let me define what I will call the reduced gravitational (Schwarzschild) “running” Planck constant $\bar{\hbar}_G$ as follows,

$$\bar{\hbar}_G(M) \equiv \frac{GM^2}{c}. \quad (4.15)$$

One trivially shows that the ordinary reduced Planck constant $\bar{\hbar}$ corresponds to the gravitational one evaluated at the Planck mass,

$$\bar{\hbar}_G(M_P) = \bar{\hbar}. \quad (4.16)$$

We now get the following ratios,

$$\frac{R_C}{R_G} = \frac{M_P^2}{M^2} = \frac{h}{h_G(M)}. \quad (4.17)$$

Obviously, quantum effects become appreciable in the region $\bar{\hbar} \gg \bar{\hbar}_G(M)$ while the classical limit corresponds to the region $\bar{\hbar} \ll \bar{\hbar}_G(M)$. In this region however, the effective Compton radius lies far inside the gravitational Schwarzschild radius and so the resonance is a black hole (or black object) with a mass much greater than the Planck mass.

In fact this region is completely analogous to the trans-Planckian region of string scattering in the eikonal approximation. There, the black hole mass $M$ is replaced by the center of mass energy, which is given by the Mandelstam variable $\sqrt{s} = 2E/c = Mc$. The classical limit then corresponds to the following trans-Planckian region $h \ll Gs/c^3$. However, at very high energy string scattering, the impact parameter plays an important role as well. It is known that classical black hole horizon formation seems to occur for very small impact parameter, at distances of the order of the classical Schwarzschild radius or less, where the eikonal approximation is valid. Various other regimes are found at higher distances while, at very large impact parameter, string theory in the usual Born (tree level) approximation gives an acceptable description of the scattering process and no black hole event horizon formation occurs.

5 Concluding remarks

When one stops and really allows for common sense to take hold for a second, it is just striking how weird it would be for statistical physics to become more fundamental than quantum physics at the level of the Planck scale. This would be a complete violation of all the known laws of physics! It was a fundamental error to believe that Statistical Mechanics in the form of Black Hole Thermodynamics could ever constitute a fundamental theory. Statistical Mechanics is a mesoscopic or macroscopic scale theory aiming at providing a physically sensible bulk description of many-body quantum systems. It does not have the ambition to explain the fundamental nature of the quantum world, by construction.
Thus far, we have established important conclusions regarding quantum black holes. Let me summarize them.

(1) Bekenstein-Hawking Black Hole Thermodynamics is an inconsistent theory because of the existence of a negative heat capacity, which leads to the impossibility of thermal equilibrium. This implies that the traditional interpretation of the black hole instanton period as an inverse canonical temperature is wrong. It further implies that no concept of black hole canonical entropy can be defined for a single quantum black hole or black object, because the canonical partition function is badly divergent, as shown immediately by taking the Laplace transform of the Hawking density of states. A single quantum black hole cannot be a thermal object. The laws of physics will not allow it.

(2) As a direct consequence, the Holographic Principle [23], based on Hawking’s area law, becomes immediately invalid. There is no connection between information and geometry in Quantum Gravity because there is no valid area law to start with, since there is no valid concept of entropy whatsoever in quantum black hole physics.

(3) Interpreting the WKB formula properly as the decay probability of the false black hole vacuum, the usual interpretation in Quantum Field Theory, the Hawking thermal black hole theory is then replaced by the pure state Leblanc-Harms theory which identifies quantum black holes and black objects as pure state quantum $p$-brane excitations at the Planck scale in various spacetime dimensions. This alternative interpretation of the same WKB formula is in agreement with general arguments stating that a Planck scale massive particle excitation should be a black hole or black object because its gravitational radius is equal or larger than its effective Compton radius.

(4) Massive string or $p$-brane excitations truly describe quantum resonances and so a consistent quantization of such theories should involve the use of so-called rigged Hilbert spaces. In this context, the massive Planck scale resonant excitations, which truly are quantum black holes, should no longer be treated as asymptotic states in string or $p$-brane theories but more properly as Planck scale Gamow (unstable) states $[76, 1, 58, 59]$. This would likely get rid of the tachyon problem.

Consequently, the definition of a quantum black hole should not be restricted to Planck scale resonant states with degeneracy of states in agreement with the semiclassical results for known classical black hole solutions to Einstein’s field equations. Instead, quantum black holes should be properly defined to include all excitations with a mass greater than the Planck mass, or equivalently with a gravitational radius larger than the effective Compton radius, whether or not they correspond to black hole (or black object) solutions to Einstein’s equations. Under this general definition, Planck scale string excitations are quantum black holes as well, although they apparently cannot be identified with known classical black holes. Quantum black holes do not have classical horizons. A classical horizon structure may be recovered in the so-called trans-Planckian classical limit.

Because quantum black holes (Planck scale $p$-brane excitations) are highly degenerate pure Gamow states $[76, 1, 58, 59]$, the statistical mechanics of a gas of such objects can only be understood within the microcanonical ensemble $[1, 2, 3, 4, 5, 6]$. Black hole systems belong to the class of so-called small systems and have negative microcanonical heat capacity. Their equilibrium state is far from thermal equilibrium, with a very inhomogeneous energy distribution.
among its constituents. Black hole gases settle in a state where almost all of the total energy (mass) is found on a single supermassive black hole, leaving almost nothing to the remnant majority. This very inhomogeneous distribution of wealth among the black holes also takes place with regards to other conserved quantities such as electric charge or angular momentum [1,2,3,4,5,6].

At the turn of the millennium however, the Russian collaboration of A. M. Fedotov, V. D. Mur, N. B. Narozhny, Vladimir A. Belinski, B. M. Karnakov and also E. G. Gel’fer published a series of serious analyses on important mathematical issues regarding the Unruh quantization procedure for accelerated observers in Minkowski spacetime [77,78,79]. These authors demonstrated rigorously that field quantization in Minkowski spacetime with the use of the so-called left and right Unruh modes leads to an inconsistency. It was shown that the Unruh modes did not form a complete set in Minkowski space, but only in the right and left Rindler wedges of that space, excluding the light-cone vertex (the origin) as well as the whole light-cone boundary. In other words, the Unruh quantization is consistent only in this double-wedge and only under the zero boundary condition for the field on the light-cone. But such a boundary condition destroys translational invariance, making it impossible for the Minkowski vacuum to be the physical vacuum of the theory. The Minkowski vacuum is translationally invariant and therefore does not admit such a zero boundary condition. Consequently the Unruh scheme has nothing to do with the question of what is seen by an accelerated observer moving through the Minkowski spacetime. The vacuum of this problem cannot correspond to the Minkowski vacuum.

Some authors [80,81] have tried to argue that the origin (and light-cone boundary) should simply be cut out of the physical spacetime, but this would no longer constitute Minkowski spacetime. In Minkowski spacetime, the Unruh effect does not exist.

Analogously, in the context of massless particles scattering off a Schwarzschild black hole background, Vladimir Belinski [82,83] of the University of Rome “La Sapienza” invoked similar arguments to demonstrate the non-existence of the related Hawking black hole evaporation effect. Belinski approached the problem by studying the case of the gravitational collapse of an infinitely thin light spherical shell with Schwarzschild metric outside and flat spacetime inside the shell.

At the beginning of the collapse, the shell radius is infinite and all of spacetime is flat both inside and outside the shell. In this case, field quantization of the initial massless radiation is performed making use of the massless limit of the boost modes already encountered in the Unruh problem for accelerated observers. Inside the shell, the standard quantization procedure leading to the Hawking effect makes use of the modes analogous to the left and right Unruh modes, with the difference that these modes are now “left” and “right” with respect to the so-called “last ray” (the continuation of the horizon into the past). Again, as in the Unruh approach, these modes are incomplete and are inadmissible for quantization of a free field in the interior region of the shell where spacetime is flat. Quantization based on these modes implies that the field has zero boundary condition on the last ray, meaning that the initial state cannot be chosen as the Minkowski vacuum, because of the lack of translational invariance. Therefore the traditional quantization procedure used in this problem has no relation to the quantization of a field in the black hole spacetime emerging from gravitational collapse, under the condition that the field was in
the Minkowski vacuum state initially. As in the Unruh problem, particle creation resulting from gravitational collapse does not occur and so the Hawking evaporation effect does not exist. This result has been supported as well by the rigorous mathematical proof that no tunneling trajectories are to be found in this process [82, 84, 85, 86].

Clearly, under correct mathematical treatment, the Hawking effect disappears. Such a conclusion supports earlier arguments by Nikishov and Ritus [22], as well as Belinski [83].

Fundamentally, the problem can be traced to the existence of a zero-energy mode on the light-cone vertex and boundary, both in the Unruh and Hawking problems. Such a zero-mode, unlike the other modes, cannot be divided into positive and negative energy parts, in contrast to the conventional derivation where it is absent. The presence of such a mode therefore destroys the Bogoliubov transformation so crucial to the existence of both the Unruh and Hawking effects. This mode also accounts for the “missing information” which resolves the information loss paradox in Mean-Field Theory and which restores translational invariance of the Minkowski vacuum.

In line with the Leblanc-Harms WKB results, Belinski has therefore been led to the fact that there is nothing fundamentally thermal about black holes. As an additional comment, let me conjecture that string or \( p \)-brane excitations being Planck scale resonances (quantum black holes), they most likely are not elementary particles. Resonances are more similar to bound states. This would imply that Planck scale string or \( p \)-brane theories are themselves not fundamental theories of matter. They are instead very much like dual models in nuclear physics, i.e effective non local theories. Their fundamental local constituents at the Planck scale remain however unknown at present (dark matter?).

In addition to the Leblanc-Harms WKB theory [1, 2, 3, 4, 5, 6], two other approaches have since become relatively well known. They are the Vafa-Strominger microscopic theory (VSMT) [87, 88], which has been well publicized in the popular press, as well as the controversial Susskind theory (SKT) [89].

The Vafa-Strominger microscopic theory (VSMT) [87, 88] is presented as a microscopic derivation of quantum black holes degeneracy of states in an effort to validate Hawking’s area law. This approach claims to have succeeded in counting the degeneracy of states of 5-dimensional (electrically or axionically) charged extreme (supersymmetric) black holes in compactified type II or heterotic superstring theories, by counting the number of so-called topological BPS (Bogomol’nyi-Prasad-Sommerfeld) solitons states as functions of the electric or axionic charges. On the other hand, Hawking and Horowitz [16] have argued that extreme black holes should not follow an area law, similarly to the cases of acceleration horizons systems. According to them, extreme black holes should have zero entropy and consequently the Vafa-Strominger theory finds itself in disagreement with BHTD, contrary to the claims of these authors. The VSMT results are pure states degeneracies and so no entropy concept whatsoever should enter the discussion in the Vafa-Strominger approach.

As for the Susskind theory (SKT) [89], it is based for the most part on the Holographic Principle [23] which has been shown in this article to be an invalid concept.

Finally, it should be emphasized that the Holographic Principle [23] mentioned in this article and which associates to a quantum black hole one degree
of freedom per Planck area (on the horizon), is totally unrelated to the so-called AdS/CFT holographic duality (correspondance) [88]. The latter applies to weak coupling type IIB superstring or $M$-theory in spacetime geometries asymptotically anti-de Sitter times a compact space. The dual strong coupling conformal field theory is defined on the boundary of the AdS spacetime.

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